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Effects of Autocorrelation Function and Partial Autocorrelation Function in Financial Market Dynamics

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Abstract

The univariate data of the Nigerian All-Share Index (NASI) obtained from the Nigerian Stock Market (NSM), covering the period of 18 years (1990-2007), was studied and analyzed through time series to ascertain certain market dynamics. The result of the analysis in a long range dependent phenomenon showed that the autocorrelation function (ACF) and partial autocorrelation function (PACF) agreed with their theoretical concepts. The observed stochastic dynamics of the NSM has the capacity to forecast future market trends. The descriptive nature of the trading activities of the NSM is further confirmed.

Keywords: Nigerian All-Share Index, Time Series, Market Dynamics, Autocorrelation Function(ACF) and Partial Autocorrelation Function(PACF).

1.0 Introduction: Nigerian Stock Exchange

This paper considered some of the essential dynamics of financial markets such as the Nigerian Stock Exchange (NSE) which establishes some multiplicative seasonal trends of a univariate data [2]. NSE maintains an All-Share Index formulated in January 3,1984. All listings are included in the Nigerian Stock Exchange All-share index. Data on listed companies performances are published daily, weekly, monthly, quarterly and annually [24]. The theory of correlations and some introductory concepts to statistics were shown in the works by [28], [29] and [30], and [5]. Works by [27] and [26], centred on time series models. [3] emphasized on the theoretical time series autocorrelated sampling properties while [7] presented the approximate distribution of serial correlation coefficients. Some tests of hypothesis in the linear autoregressive model are shown in [10] and [11]. Some approximate tests of correlation in time series could be found in [20] and [13]. [25] and [1] respectively worked on estimation and information in stationary time series. [9] gave a bilinear representation of time series with applications by obtaining expressions for covariance while assuming that the random variables $e_t$are Gaussian with $E(e_t)$= 0. [21] measured forecast performance of ARMA and ARFIMA models with application to US dollar and UK pound.

2.0 Methodology

We consider three processes associated with ARIMA (p,d,q) models. These include:

$$z_t = c + \phi_1 z_{t-1} + a_t \qquad (2.1)$$

$$z_t = c - \theta_1 a_{t-1} - \theta_2 a_{t-2} + a_t \qquad (2.2)$$

$$z_t = c + \phi_1 z_{t-1} - \theta_1 a_{t-1} + a_t \qquad (2.3)$$

$$c = \mu\left(1 - \sum_{i=1}^{p} \phi_i\right)$$

(2.4)

Equation (2.1) is called an AR(1) process because it contains only one AR term (including the constant term and the current random shock) where the maximum time lag on the AR terms is two. Equation (2.2) is called an MA(2) since it has only MA terms with a maximum time lag on the MA terms of two. Equation (2.3) is an example of a mixed process because it contains both AR and MA terms. It is an ARIMA(1,1) with AR(1) and MA(1). Equation (2.4) gives the constant term of an ARIMA process. If no AR terms are present, then$c = \mu$. This is true for all MA processes. ARIMA processes are characterized by the values of p, d, q in this manner: ARIMA(p,d,q), where $p, d, q \in \mathbb{Z}^+$ such that p is the AR order of the process, q is the MA order of the process and d is the number of times a realization must be differenced to achieve a stationary mean. The letter "I" in the acronym ARIMA refers to the integration step which corresponds to the number of times, d, the original series has been differenced. If a series has been differenced d times, it must subsequently be integrated d times to return to its original overall level, as expressed by [19].

2.1 ARIMA Models in Back shift Notation

ARIMA models are often written in back shift notation. The back shift operator, B, alters the time subscript on the variable by which it is multiplied, that is

$$B^k z_t = z_{t-k} \qquad k = 1, 2, ..., n \qquad (2.5)$$

Also,

$$B^k C = C \forall k < \infty \qquad (2.6)$$

Multiplying z_t by the differencing operator, $(1 - B)^d$, produces the d^{th} differences of z_t:

$$(1 - B)^d z_t = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) z_t \qquad (2.7)$$

The procedure for non-seasonal processes in back shift form has six steps as seen in [19]. Using the six steps, a non-seasonal process in back shift notation has the general form:

$$(1-\emptyset_1 B-\emptyset_2 B^2-\cdots-\emptyset_P B^p)(1-B)^d\,\tilde{z}_t = \left(1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q\right)a_t \qquad (2.8)$$

where $(1-\emptyset_1 B-\emptyset_2 B^2-\cdots-\emptyset_P B^p)$ is the general form of the AR operator of order $(1-\theta_1 B-\theta B^2-\cdots-\theta_P B^p)$ is the general form of the MA operator of order q, z_t written in deviations from its mean, $\tilde{z}_t$, and a_t is the random shock. Equation (2.8) can be written in compact notation by substituting the following symbols.

Let

$$\nabla = 1-B \qquad (2.9)$$

$$\emptyset(B) = 1-\emptyset_1 B-\emptyset_2 B^2-\cdots-\emptyset_P B^q \qquad (2.10)$$

$$\theta(B) = 1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q \qquad (2.11)$$

Thus, a compact way of saying that the random variable $z_t$evolves according to an ARIMA(p,d,q) process is

$$\emptyset(B)\nabla^{\mathrm{d}}\tilde{z}_t = \theta(B)a_t \qquad (2.12)$$

It is more difficult to show that MA terms represent past $z's$. By demonstrating this for MA(1) rather than proving it for the general case, we will find out that the MA(1) process can be interpreted as an AR process of infinitely high order. MA(1) process in back shift form is:

$$\tilde{z}_t = (1-\theta_1 B)a_t \qquad (2.13)$$

$$\Longrightarrow \quad a_t = (1-\theta_1 B)^{-1}\tilde{z}_t \qquad (2.14)$$

For a geometric series with $|\theta_1| < 1, (1-\theta_1 B)^{-1}$ is the sum of a convergent infinite series, that is,

$$(1-\theta_1 B)^{-1} = 1+\theta_1 B+\theta_1^2 B^2+\theta_1^2 B^3+\cdots, \quad |\theta_1| < 1 \qquad (2.15)$$

$$\Longrightarrow a_t = (1+\theta_1 B+\theta_1^2 B^2+\theta_1^3 B^3+\cdots)\tilde{z}_t \qquad (2.16)$$

Equation (2.16) is an AR process of infinitely high order with the$\emptyset$coefficients given by

$$\left.\begin{array}{l}\phi_1 = -\theta_1\\ \phi_2 = -\theta_2\\ \phi_3 = -\theta_3\\ \vdots\end{array}\right\} \qquad (2.17)$$

2.2 The Estimated ACF and PACF.

The estimated ACF and the estimated PACF are very important tools at the identification stage of the UBJ method [6].

2.2.1 Estimated ACF

The idea in an autocorrelation analysis is to calculate a correlation coefficient for each set of ordered pairs $(\tilde{z}_t, \tilde{z}_{t+k})$. Since we need to find the correlation between sets of numbers that are part of the same series, the resulting statistic is called an autocorrelation coefficient. Let r_k be the estimated autocorrelation coefficient of observations separated by k time periods within a given series. The standard formula for calculating autocorrelation coefficients is:

$$r_k = \frac{\sum_{t=1}^{n-k}(z_t - \tilde{z})(z_{t+k} - \tilde{z})}{\sum_{t=1}^{n}(z_t - \tilde{z})^2} \qquad (2.18)$$

or more compactly as

$$r_k = \frac{\sum_{t=1}^{n-k}\tilde{z}_t \tilde{z}_{t+k}}{\sum_{t=1}^{n}\tilde{z}_t^{\,2}} \qquad (2.19)$$

where $\tilde{z}_t$ and $\tilde{z}_{t+k}$ have their usual meaning as shown.

2.2.2 Estimated PACF

An estimated PACF is broadly similar to an estimated ACF. An estimated PACF is also a graphical representation of the statistical relationship between sets of ordered pairs $(\tilde{z}_t, \tilde{z}_{t+k})$ drawn from a single time series. The estimated PACF is used as a guide, together with the estimated ACF, in choosing one or more ARIMA models that might fit the available data. The estimated partial autocorrelation coefficient measuring this relationship between $\tilde{z}_t$ and $\tilde{z}_{t+k}$ is designated by $\hat{\phi}_{kk}$. (Recall that $\hat{\phi}_{kk}$ is a statistic because it is calculated from sample information and provides an estimate of the true partial autocorrelation coefficient ϕ_{kk} *.)* The steps to obtain $\hat{\phi}_{kk}$ are as follows. Initially, we estimate the following regression

$$\tilde{z}_{t+1} = \phi_{11}\tilde{z}_t + U_{t+1} \qquad (2.20)$$

for $\hat{\phi}_{11}$ where ϕ_{11} is the true partial autocorrelation to be estimated by regression for $k = 1$, where U_{t+1} is the error term representing all things affecting $\tilde{z}_{t+1}$ that do not appear elsewhere in the regression equation. Using least squares regression computer program, we obtain $\hat{\phi}_{11}$ for $k = 1$. To obtain, $\hat{\phi}_{22}$ we have to estimate the multiple regression:

$$\tilde{z}_{t+2} = \phi_{21}\tilde{z}_{t+1} + \phi_{22}\tilde{z}_t + U_{t+2} \tag{2.21}$$

where ϕ_{22} is the true partial autocorrelation coefficient to be estimated for $k = 2$.Therefore, $\hat{\phi}_{22}$ estimates the relationship between $\tilde{z}_t$ and $\tilde{z}_{t+1}$ with $\tilde{z}_{t+1}$ accounted for. Next, we estimate the following regression

$$\tilde{z}_{t+3} = \phi_{31}\tilde{z}_{t+2} + \phi_{32}\tilde{z}_{t+1} + \phi_{33}\tilde{z}_t + U_{t+3} \tag{2.22}$$

to find $\hat{\phi}_{33}$ where $\hat{\phi}_{33}$ is the partial autocorrelation coefficient to be estimated for $k = 3$. Thus, $\hat{\phi}_{33}$ estimates the relationship between $\tilde{z}_t$ and $\tilde{z}_{t+3}$ with $\tilde{z}_{t+1}$ and $\tilde{z}_{t+2}$ accounted for. There is a slightly less accurate though computationally easier way to estimate the ϕ_{kk} coefficients. It involves using the previously calculated autocorrelation coefficientsr_k. If the data series is stationary, then the following set of recursive equations gives fairly good estimates of the partial autocorrelations.

$$\hat{\phi}_{11} = r_1$$

and

$$\hat{\phi}_{kk} = \frac{r_k - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_{k-j}}{1 - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_j} \qquad (k = 2,3,\dots) \tag{2.23}$$

where $\hat{\phi}_{kj} = \hat{\phi}_{k-1,j} - \hat{\phi}_{kk}\hat{\phi}_{k-1,k-j}\,(k = 3,4,\dots,; \quad j = 1,2,\dots,k-1$

2.3 Long-Range Dependence

Definition 2.1 A stationary sequence $(X_n)_{n\in\mathbb{N}}$ exhibits long-range dependence if the autocovariance function $\rho(n) := cov(X_k, X_{k+n})$ satisfy

$$\lim_{n\to\infty} \frac{\rho(n)}{cn^{-\alpha}} = 1 \tag{2.24a}$$

for some constant c and $\alpha \in (0,1)$. In this case, the dependence between X_k and X_{k+n}decays slowly $as\ n \longrightarrow \infty$ and

$$\sum_{n=1}^{\infty} \rho(n) = \infty \tag{2.24b}$$

Hence, according to [4], we obtain immediately that the increments.

$$X_k := B_k^H - B_{k-1}^H \tag{2.25}$$

and

$$X_{k+n} := B_{k+n}^H - B_{k-1}^H \tag{2.26}$$

of B^H have the long-range dependence property for the Hurst parameter, $H > \frac{1}{2}$ since

$$\rho_H(n) = \frac{1}{2}[(n+1)^{2H} + (N-1)^{2H} - 2n^{2H}] \sim H(2H-1)n^{2H} asn \to \infty \tag{2.27}$$

In particular,

$$\lim_{n\to\infty} \frac{\rho_H(n)}{H(2H-1)n} = 1 \tag{2.28a}$$

Summarizing, we obtain:

$$\sum_{n=1}^{\infty} \rho_H(n) = \infty, \qquad H > \frac{1}{2} \tag{2.28b}$$

$$\sum_{n=1}^{\infty} |\rho_H(n)| < \infty, \qquad H < \frac{1}{2} \tag{2.28c}$$

There are alternative definitions of long-range dependence. We recall that a function L, is slowly varying at zero (respectively, at infinity) if it is bounded on a finite interval and if for all $\alpha > 0$, $\frac{L(\alpha x)}{L(x)} \to 1\ as\, n \to \infty$ respectively. The spectral density of the autocovariance $\rho(k)$ is given by

$$f(\lambda) := \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-i\lambda k} \rho(k) \quad \forall \lambda \in [-\pi, \pi] \tag{2.29}$$

Definition 2.2 For stationary sequences, $(X_n)n\epsilon\mathbb{N}$, with finite variance, we say that $(X_n)_{n\in\mathbb{N}}$ exhibits long-range dependence if one of the followings holds:

For some constants c *and* $\beta \in (0,1)$,

$$\lim_{n\to\infty} \sum_{k=-n}^{\infty} \rho(k) / cn^{\beta} L_1(n) = 1 \tag{2.30a}$$

For some constant c *and* $\gamma \in (0,1)$,

$$\lim_{n\to\infty} \rho(k) / ck^{-\gamma} L_2(k) = 1 \tag{2.30b}$$

For some constant c *and* $\delta \in (0,1)$,

$$\lim_{\lambda\to\infty} f(\lambda) / c|\lambda|^{-\delta} L_3|\lambda| = 1 \tag{2.30c}$$

where L_1, L_2 are slowly varying functions at infinity, while L_3 is slowly varying at zero.

Lemma 2.1. For fractional Brownian motion, (fBm), B_t^H of Hurst index , $H \in (\frac{1}{2}, 1)$, the three definitions of long-range dependence of Definition (2.2) are equivalent. They hold with the following choice of parameters and slowly varying functions [3]:

$$\beta = 2H - 1, L_1(x) = 2H \tag{2.31a}$$

$$\gamma = 2 - 2H,\ L_2(x) = H(2H - 1) \tag{2.31b}$$

$$\delta = 2H - 1,\ L_3(x) = \pi^{-1} H\Gamma(2H) sin\pi H \tag{2.31c}$$

Proof. See [4], [23] and [8].

Definition 2.3 We say that an $\mathbb{R}^d$ - *valued* random process $X = (X_t)_{t\geq 0}$, is self-similar or satisfies the property of self-similarity if for every $a > 0$ there exists $b > 0$ such that:

$$Law(X_{at,} t \geq 0) = Law(bX_{t,} t \geq 0 \tag{2.32}$$

Hence the two processes X_{at} and bX_t have the same finite-dimensional distribution functions, i.e. $\forall t_{0,} \ldots t_n \in \mathbb{R}$, ([4] and [22])

$$P(X_{at_0} \leq x_0, \ldots, X_{at_n} \leq x_n) = P(bX_{t_0} \leq x_0, \ldots, bX_{t_n} \leq x_n) \quad \forall x_{i,} i, \ldots, n \in \square \tag{2.33}$$

3.0 Data Analysis

The seasonal decomposition on Nigerian All-Share Index (NASI) data from 1990 to 2007 [24], is presented in this section in order to observe some trends. Through the computer simulation of the seasonal decomposition of NASI, we find two trends. The first one is multiplicative type with MULT as trend name and NASI MULT as serial name while the second is additive type with

ADDT as trend name and NASI ADDT as serial name. The MULT is chosen because its average periodicity percentage growth rate observed is better than that of ADDT [2].

3.1 Multiplicative Trend: NASI MULT

The seasonal multiplicative trend type, MULT, with serial name, NASI MULT, has length of seasonal period 4. In computing the method of moving averages, the observations span is equal to the periodicity and all points are weighted equally. Applying the model specifications from MULT, the seasonal factors (%)for four periods are: Period 1,99.2%; period 2,100.5%; period 3,100.6% and period 4, 99.7%.

3.2 Additive Trend: NASI ADDT

The seasonal additive trend type, ADDT, with serial name, NASI ADDT, has seasonal length 4. In computing the method of moving averages, the observations span equal to the periodicity, and all points weighted equally. Also the seasonal factor (%) for four periods are: period 1, 1889.446%; period 2,1042.8535%; period 3, 823.49111% and period 4, 23.12095%. However, the seasonal multiplicative trend type, NASI MULT, was chosen because it has a higher average periodicity percentage growth than NASI ADDT [2].

3.3 NASI MULT of ARIMA(1,1,1)(0,0,0)

The time series modeler trend type of NASI MULT analysed was ARIMA (1,1,1)(0,0,0) with the model statistic summary chart given in Table 3. The Portmanteau test (Ljung-Box Q(18)) is for 18 years (1990-2007). According to [12], the Ljung-Box Test for lack of fit is a diagnostic tool used to test the lack of fit of a time series model. The test is applied to the residuals of a time series after fitting an ARIMA (p,q) model or ARIMA(p,d,q) to the data. The test examines auto-correlations of the residuals. If the autocorrelations are very small, we conclude that the model does not exhibit significant lack of fit. The Ljung-Box test is implemented using a residual time series, see Ljung-Box test website for more details. In general, the Ljung-Box test is defined as:

H_0: The model does not exhibit lack of fit;

H_a: The model exhibits lack of fit.

Test Statistic: Given a time series Y of length n, the test statistic is defined as:

$$Q = n(n+2)\sum_{k=1}^{m}\frac{\hat{r}_k^2}{n-k}$$

where $\hat{r}_k$ is the estimated autocorrelation of the series at lag k, and m is the number of lags being tested. For significance level a, and the critical region, the Ljung-Box test rejects the null hypothesis (indicating that the model has significant lack of fit) if $Q > X_{1-\alpha,h}^2$ which gives the Chi-square distribution table value with h degrees of freedom and significance level α.Because the test can be applied to residuals, the degrees of freedom must account for the estimated model parameters so that $h = m - p - q$, where p and q indicate the number of parameters from the ARIMA(p,d,q) model fit to the data.

Table 1: The Autocorrelations of the NASI MULT

Lag	Autocorrelation	Standard Error[a]	Box-Ljung Statistics		
			Value	df	Sig[b]
1	.879	.115	57.981	1	.000
2	.764	.115	102.374	2	.000
3	.659	.114	135.952	3	.000
4	.628	.113	166.843	4	.000
5	.569	.112	192.547	5	.000
6	.521	.111	214.455	6	.000
7	.495	.110	234.511	7	.000
8	.467	.110	252.643	8	.000
9	.435	.109	268.682	9	.000
10	.414	.108	283.383	10	.000
11	.393	.107	296.863	11	.000
12	.373	.106	309.194	12	.000
13	.341	.105	319.697	13	.000
14	.297	.104	327.773	14	.000
15	.230	.103	332.720	15	.000
16	.175	.103	335.627	16	.000

a. The underlying process assumed is independence (white noise);
b. Based on the asymptotic Chi-square approximation.

Table 2: PACF of NASI MULT

Lag	Partial Autocorrelation	Standard Error
1	.879	.118
2	-.039	.118
3	-.016	.118
4	.259	.118
5	-.142	.118
6	.041	.118
7	.157	.118
8	-.104	.118
9	.030	.118
10	.101	.118
11	-.080	.118
12	.035	.118
13	-.006	.118
14	-.138	.118
15	-.097	.118
16	.012	.118

Table 3: Model Statistic of ARIMA(1,1,1) of NASI MULT

Model	Number of Predictors	Model Fit Statistic		Ljung-Box *Q(18)*			Number of Outliers
		Stationary R-squared	R-squared	Statistics	DF	Sig.	
NASI MULT	0	.006	.942	10.266	16	.852	0

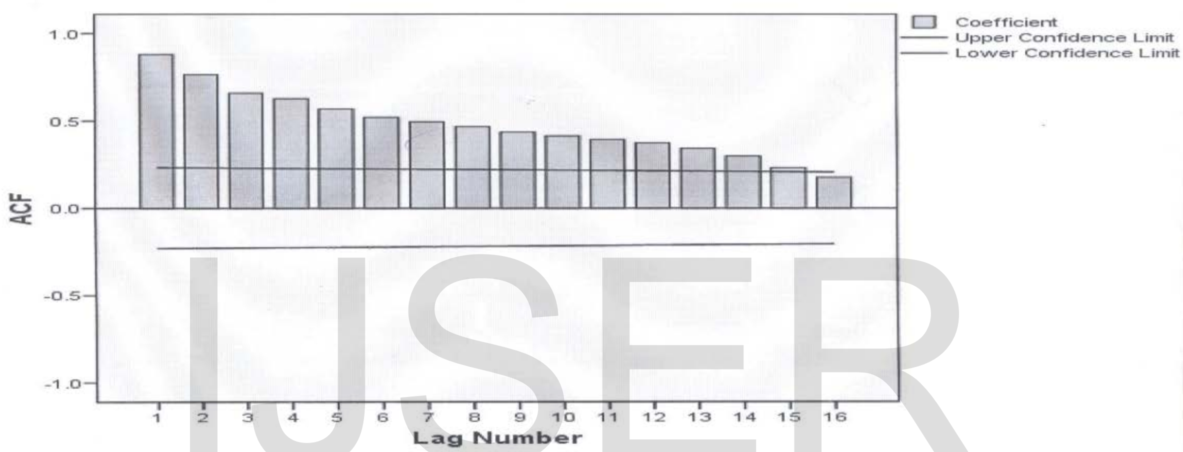

Figure1: The ACF Plot of the NASI, showing the autocorrelation coefficients

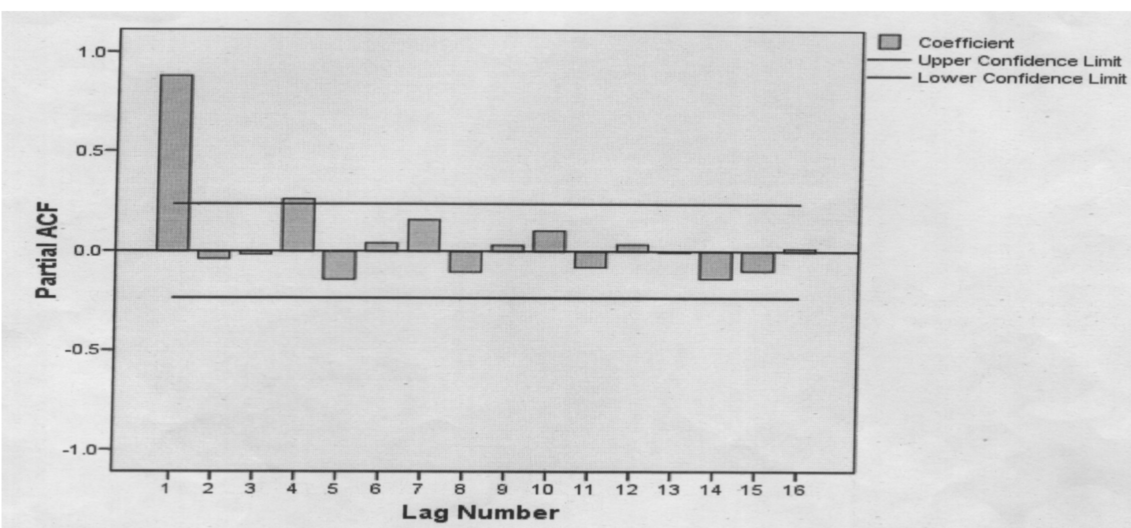

Figure 2: The PACF Plot of the NASI, showing the Partial autocorrelation coefficients

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4.0 Result

Figure 1 is the plot of Table 1 which showed the autocorrelations of the NASI MULT with 16 lags. Similarly, Figure 2 is the corresponding plot of Table 2 which showed the partial autocorrelations of the NASI MULT with 16 lags. Table 3 showed the model statistic for ARIMA(1,1,1) of NASI MULT with zero values for both predictors and outliers. The model fit statistic has stationary R-squared value as 0.006, and R-squared value as 0.942, while Ljung-Box *Q(18)* statistics, degree of freedom and significance level are 10.266, 16 and 0.852 respectively.

Conclusion

Figures 1 and 2 showed the outcome of seasonal decomposition on the NASI. Since the more general ARFIMA (p,d,q) process can include short memory autoregressive (AR) or moving average (MA) processes over a long memory process, it has potential in describing markets. Hence, the result of the analysis in a long range dependent phenomenon showed that the ACF and PACF agreed with their theoretical concepts. The observed dynamics of the NSM has the capacity to describe the market trends in terms of some of its characteristic exponents (e.g. H), correlation between (stochastic) increments, fractional noise effect and residuals. The descriptive nature of the trading activities of the NSM is further confirmed as shown in Tables and Figures.

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